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SELF-OSCILLATION REGIMES IN A SYSTEM OF FOUR QUASI-TWO-DIMENSIONAL VORTICES

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UDC 532.51

Particular attention has recently been devoted to experimental studies of transitional processes in the appearance of turbulence in simple hydrodynamic flows. In the present study we present a model of the elementary cell of a quasi-two-dimensional double-period flow that is related to Kolmogorov flow [1-4]. The derived results may prove to be useful, for example, in application to the problem of constructing limited-mode systems which, in basic outline, describe the nonlinear processes occurring in hydrodynamic flows [2, 3].

The primary flow regime is a steady system of four quasi-two-dimensional vortices. The self-oscillations in such a system were first detected in studies of the convective motion in a Haley—Shaw cell [5, 6], and subsequently in a uniform fluid in which the flow was induced by means of a magnetohydrodynamic drive [7-9].

It is the aim of the present study to further investigate the above-indicated system of vortices. The flow is generated in a horizontal rectangular cuvette in layers of various thicknesses, under the action of an MHD force periodic along both coordinates. In particular, we have derived the relationship between the amplitudes of the self-oscillations and the Reynolds number, and a spectral analysis of the self-oscillation regimes has been carried out. We have examined the effect of friction against the bottom on the characteristics of the flow.

1. Laboratory Equipment and the Experimental Method. The experiments were conducted on an installation such as that described in [9]. The flow was established within a rectangular cuvette having dimensions of 24 × 12 × 3 cm, positioned horizontally on a Plexiglas frame. Two three-pole electromagnets are contained symmetrically within the frame. The magnetic field induction **B** of the electromagnets within the region of the cuvette has a vertical component which can approximately be presented in the form

$$B_z(x, y, z) = B_0(z) \sin (2\pi x/L_x) \cos (2\pi y/L_y).$$

Here $B_0(z)$ is the amplitude $B_z(x, y, z)$ on the plane z = const; L_x = 24 cm and L_y = 12 cm represent the length and width of the cuvette along the x and y axes, lying on the plane z = 0 and coincident with the two adjacent sides of the cuvette. The z axis is directed vertically upward. An electrolyte (a $CuSO_4$ solution with a concentration of 100 g/liter)

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is poured into the cuvette. An electric current of density $\mathbf{j} = (0, j_y, 0)$ is passed between the copper electrodes set flush into the side walls of the cuvette, and an Ampere force $\mathbf{F} = \rho^{-1}c^{-1}[\mathbf{j}, \mathbf{B}]$ acts on each unit of fluid mass (ρ is the density of the electrolyte, and c is the electrodynamic constant equal to the speed of light in a vacuum). The flow is defined virtually exclusively by the component $F_x = \rho^{-1}c^{-1}j_yB_z$, since $F_y = 0$, and the Reynolds number Re in accordance with $F_z = \rho^{-1}c^{-1}j_yB_x$ in the experiments is smaller approximately by four orders of magnitude than the corresponding Re that is based on F_x (see Sec. 3). Moreover, $F_z \lesssim 10^{-4}$ g, where g is the acceleration due to gravity. The quantity $B_0(z)$ changes in accordance with a law that is nearly exponential. Its value in the calculations is taken at an average level for this electrolyte layer. The electromagnet is connected to a laboratory stabilized TES-18 rectifier which maintains a constant current strength of 1000 ± 3 mA in the windings. The required power amounts to 11 W. The cooling system is based on a U10 ultrathermostat, connected to an external water conduit; transformer oil was used as the cooling fluid. The temperature of the electrolyte during a single experiment was kept constant to within ±0.1°C and monitored by means of copper—constantan thermocouple. Power was supplied to the cuvette from a laboratory stabilized TES-20 rectifier. The cuvette was closed at the top by means of a Plexiglas lid.

The results presented here were obtained from measurements of the difference between the flow velocities at two points symmetrical relative to the $y = L_y/2$ plane by means of a thermoanemometer based on two identical MT-54M microthermoresistors, connected in parallel to the measurement bridge. The thermoresistors were immersed to a depth of 3 mm beneath the surface of the fluid at points having the following coordinates (7.6 cm, 1.2 cm) and (7.6 cm, 10.8 cm). These points were chosen so that the Re range within which the flow velocity does not change sign will be at its maximum. For the given points this requirement is correct for $Re \leq 3Re^*$. The streamlining of the sensors is steady ($Re \sim 10$). This method makes it possible, first of all, to isolate fluctuations in velocity relative to some average value and, secondly, ensures, as demonstrated by estimates, a virtually linear characteristic for the thermoanemometer. This is associated with the fact that in this particular range of Re numbers, together with the increase in velocity at one of these points in the case of self-oscillations, there is a reduction in velocity that is approximately the same magnitude as in the other. The nonlinearity of the characteristics at these points of the thermoresistors is compensated. The signal taken from the measurement bridge is fed through an amplifier to the input of a cassette recorder [11]. A S1-82 oscillograph is used to monitor the recordings. The resulting information is subsequently processed on a computer. A typical recording consists of $12 \cdot 10^4$ points at a quantification frequency of 120 Hz. There was a parallel recording of the signal on the tape of an N3030-1 electrometer. Visual observations of the flow were also carried out. A similar experimental method was employed in [4, 9].

2. Dimensionless Flow Parameters. The flow being investigated here has three determining dimensionless parameters: a) the Reynolds number "based on the external force" $Re = (L_x/2\pi)^3(j_yB_0/\rho c \nu^2)$ (ν is the coefficient of kinematic viscosity). The Re value was monitored to an accuracy of ~1%; b) the dimensionless bottom-friction parameter $\sigma_0 = (L_x/h)^2$; c) the dimensionless geometric parameter (eccentricity) of the cuvette $\varepsilon = L_xL_y$. The experiments were carried out at $Re \leq 3Re^*$ in nine different fluid layers with $122.7 \leq \sigma_0 \leq 1963$ in a cuvette with $\varepsilon = 2$.

Let us introduce yet another dimensionless quantity, namely, the Reynolds number based on the "external friction" parameter $\lambda = \nu/h^2$ [10]: $Re_\lambda = (h^4/8\pi^3L_x)(j_yB_0/\rho c\nu^2)$. This can be used as one of the determining parameters ($Re_\lambda = Re/\sigma_0{}^2$). In discussing the experimental results we will make use of the general time scale $\tau_L = L_x{}^2/\nu$ and the time scale $\tau_h = h^2/\nu$ obtained through the thickness of the layer (the attenuation time). The frequency values will be presented in two forms, respectively: $f_L = L_x{}^2/T\nu$ and $f_h = h^2/T\nu$ (T is the measured period of self-oscillations). The average τ_L over all of the experiments is equal to $5 \cdot 10^4$ sec.

3. Primary Steady Regime and the Excitation of Self-Oscillations. With small Re a primary regime is formed within the cuvette, which is a steady system of four quasi-two-dimensional vortices (Fig. 1a). The vortices positioned on one of the diagonals exhibit identical rotational direction. As the values of Re pass through the critical value $Re^* = Re^*(\sigma_0)$ in the flow, self-oscillations are excited in the soft regime. Since the critical value of Re depends significantly on σ_0, the supercriticality of the flow is conveniently characterized by the ratio Re/Re^*. With $(Re/Re^*) \sim 1$ the self-oscillations are monochromatic.

Within the self-oscillation period T the vortices with identical sign undergo one closure, while the total vortex value through the entire flow remains virtually zero [9]. With an increase in Re/Re* the frequency and amplitude of the self-oscillations increase, and the higher harmonics are excited within the flow, i.e., the spatial structure of the flow is made more complex. Figure 1b-e shows a motion-picture record of the self-oscillations ($t = t_0$, $t_0 + T/10$, $t_0 + T/5$, $t_0 + T/2$) in a flow with $\sigma_0 = 843$ for (Re/Re*) = 3.1. As in the (Re/Re*) ~ 1 case, we observe cyclical closure of vortices with identical sign, i.e., a complex spatial flow structure is clearly replicated from period to period. The total vortex value, estimated from the tracks, is significantly different from zero and periodically changes sign. Let us note that in the narrow Re range between the primary steady and self-oscillatory regimes a secondary three-vortex steady regime is possibly realized [8], but a detailed study of this regime goes beyond the scope of this paper. The values of Re* were determined from "quasistatic" increments of Re, i.e., the value of Re increased by 2-5% and then some time was permitted to elapse, during which the velocity field was adapted to a new Re value. For Re ≫ Re* this adaptation time amounts to several minutes, while in the case of Re ~ Re* it involves several hours (depending on σ_0). The larger σ_0 (the thinner the fluid layer) the more dynamic the processes and the smaller the waiting times. For purposes of these measurements the required Re was reached through "slow" growth. The difference between this method and the "quasistatic" method lies in the fact that the waiting time for both Re ≪ Re* and Re ≳ Re* amounted to 3-5 min. With such an application of the external force we estimated the incremental value γ of the oscillations in the weakly supercritical flow. The quantity γ for a fixed σ_0 increases as Re/Re* increases. With a fixed Re/Re*, γ increases as σ_0 increases. The typical values of γ are on the order of 10^{-3} sec^{-1}. For example, in a flow with $\sigma_0 = 122.7 \pm 0.4$ for (Re/Re*) = 1.18 ± 0.03 we have $\gamma = (0.30 \pm 0.02)10^{-3}$ sec^{-1}; with $\sigma_0 = 843 \pm 3$ and (Re/Re*) = 1.06 ± 0.03, $\gamma = (1.17 \pm 0.04)10^{-3}$ sec^{-1}. The cited values of γ were derived in a time segment in which the average magnitude of the oscillation amplitude was smaller by a factor of 4-7 than that which was finally established (in the first case, within 3 h, and within 1 h in the second case).

One additional method of exciting the flow was utilized, namely, the required Re value was established through impact application of the external force. Here it was observed that subcritical attenuating self-oscillations exist. The self-oscillations are excited when Re < Re* (Re ~ Re*) at virtually the same time as the application of the external force and within a time equal to 3-5 periods and more these are attenuated. The self-oscillation attenuation times are all the greater, the smaller Re − Re* and σ_0. If Re > Re* (Re ~ Re*)

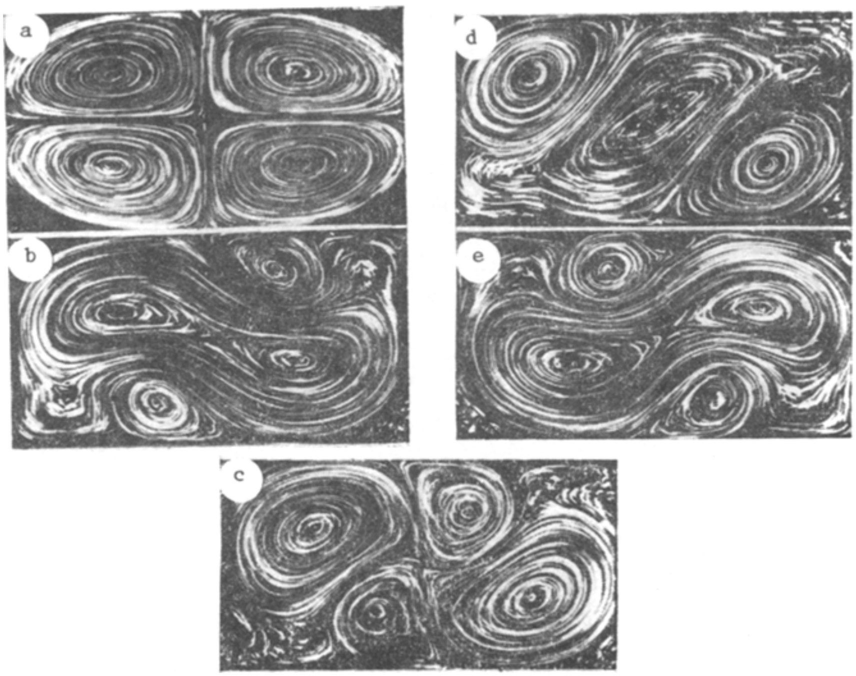

Fig. 1

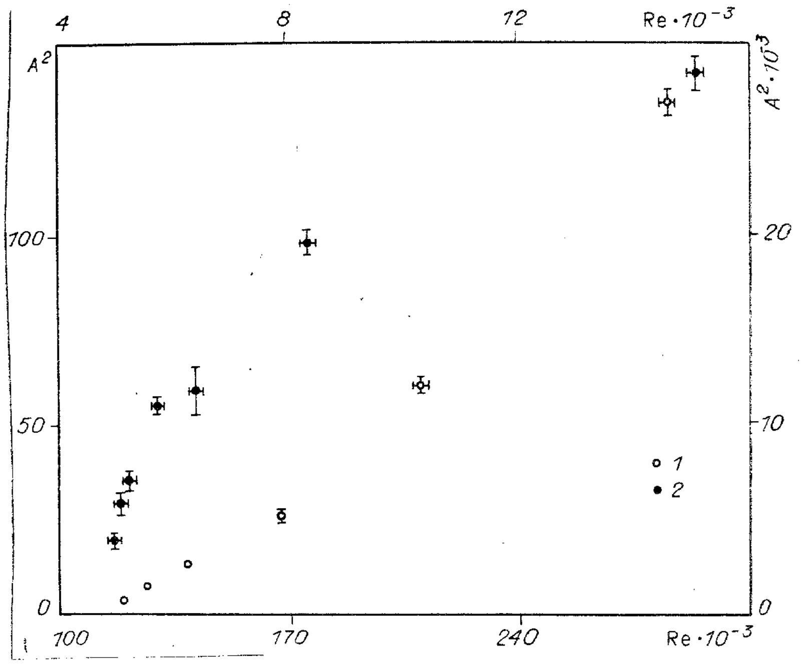

Fig. 2

is established through impact, the self-oscillations are also excited and attenuated, but within some period of time they are reexcited. With larger Re − Re* a similar pattern is observed; however, the oscillations are not completely attenuated and their amplitude from some nonzero level begins to grow, reaching its characteristic value. The impact application of the external force may, apparently, be regarded as the introduction into the steady flow of spatially periodic perturbations of finite amplitude.

4. Fundamental Characteristics of the Self-Oscillations. With small Re − Re* for the amplitude of the signal from the thermoanemometer, proportional to the difference between the flow velocities at two fixed symmetric points, the following root law [12] is satisfied:

$$A = k(\mathrm{Re} - \mathrm{Re}^*)^{1/2}.$$

The coefficient $k = k(\sigma_0)$ diminishes as σ_0 increases. With a further increase in Re − Re* the higher harmonics are established in the flow spectrum, and a qualitative difference in the behavior of the function $A^2(\mathrm{Re})$ is observed in the layers with $\sigma_0 \leq 497$ and $\sigma_0 \geq 843$. When $\sigma_0 \leq 497$ the experimental points fall out on the curve with the downward bulge, while when $\sigma_0 \geq 843$ they fall out on the curve with the upward bulge. This is illustrated in Fig. 2, where the obtained points in the layer with $\sigma_0 = 122.7 \pm 0.4$ are identified with the numeral 1, while the points found in the layer with $\sigma_0 = 1110 \pm 4$ are marked with the numeral 2. In the first case the values of A^2 have been plotted along the right-hand side of the ordinate axis and the values of Re have been plotted along the upper axis of abscissas, while in the second case, these values have, respectively, been plotted on the left-hand ordinate axis and on the lower axis of abscissas. The foregoing pertains to the range $1.05\mathrm{Re}^* \leq \mathrm{Re} \leq 3\mathrm{Re}^*$, with no reliable data available for $\mathrm{Re} < 1.05\mathrm{Re}^*$. We note that with an identical value for Re/Re* the velocity throughout the surface of the layer rises noticeably as σ_0 increases (with the exception of the centers of the vortices and the hyperbolic point). Thus, when $(\mathrm{Re}/\mathrm{Re}^*) = 1$ for the component **u** along the x axis at the points where the probes are located in layers with σ_0 = 497, 843, and 1963 we have, respectively, u_x = 0.45 ± 0.04 cm/sec, 0.73 ± 0.07 cm/sec, and 1.5 ± 0.2 cm/sec. The period T/τ_L of the self-oscillations diminishes monotonically as Re increases for all σ_0. The smaller σ_0, the larger the relative change T/τ_L for the same change in Re/Re*. For fixed Re/Re*, T/τ_L increases with a reduction in σ_0 [9].

Figure 3 shows the functions $\mathrm{Re}^*(\sigma_0)$, $\mathrm{Re}_\lambda^*(\sigma_0)$, $f_L(\sigma_0)$, and $f_h(\sigma_0)$ constructed on the basis of the experimental points. The cited values of $f_L(\sigma_0)$ and $f_h(\sigma_0)$ were derived for $(\mathrm{Re}/\mathrm{Re}^*) = 1.5$. The values of $\mathrm{Re}^*(\sigma_0)$ and $f_L(\sigma_0)$ increase with an increase in σ_0; $\mathrm{Re}_\lambda^*(\sigma_0)$ and $f_h(\sigma_0)$ diminish noticeably with an increase in σ_0 (when $\sigma_0 \leq 497$) and change weakly when $\sigma_0 \geq 843$. It might be assumed that in the interval $497 < \sigma_0 < 843$ each of the functions has either a breaking point or two breaking points. Let us also note that in layers with $\sigma_0 \geq 843$ it may be possible, in one sense or another, to speak of the self-similarity of the flow in terms of σ_0. The flow being examined here, given limited dimensionless thickness of the layer, essentially becomes a single-parameter flow and is characterized exclusively by Re_λ. With regard to the quasi-two-dimensional shearing flows, the determining parameters are Re and Re_λ and the self-similarity of the flow has been demonstrated in [10] for thin layers in terms of Re.

With $(Re/Re^*) \sim 1$ the time dependence of the signal from the thermoanemometer is nearly sinusoidal and becomes more complex as Re/Re^* increases. Throughout the entire $Re \leq 3Re^*$ range within the limits of sensitivity for the electrometer the shape of the self-oscillations is reproduced in all details in Fig. 4a-d, i.e., $(Re/Re^*) = 1.13$, 3.18, 2.85, and 3.85 . Given the same value of Re/Re^* the shape of the oscillations markedly becomes more complex as σ_0 increases, but changes weakly with a change in σ_0 within each of the intervals (Fig. 4a, b, i.e., $\sigma_0 \doteq 497$, and in Fig. 4c, d, in the case where $\sigma_0 = 843$ and 1963). With fixed Re/Re^* the number of harmonics excited in the flow also increases as σ_0 increases. Figure 5a-d shows the time spectra of the self-oscillations in the layer with $\sigma_0 = 843$ for $(Re/Re^*) = 1.06$, 1.25, 2.12, and 2.98. The corresponding values of the clear factor (the harmonic coefficient) are equal to 0.016, 0.036, 0.26, and 0.38. In the layers with $\sigma_0 \leq 497$, given the same Re/Re^*, the number of overtones excited in the flow is significantly smaller. For example, with $(Re/Re^*) \sim 3$ in a flow with $\sigma_0 = 497$ only 5-6 higher harmonics are excited.

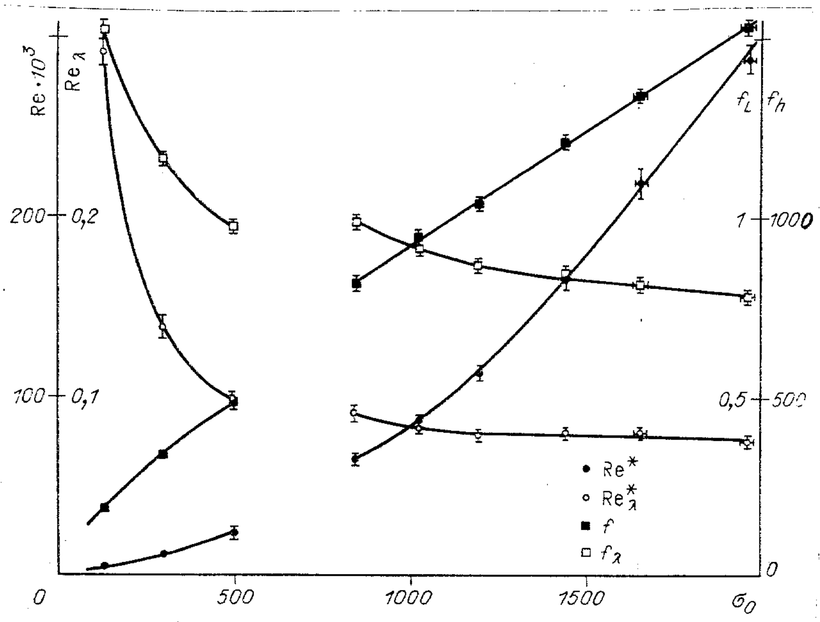

Fig. 3

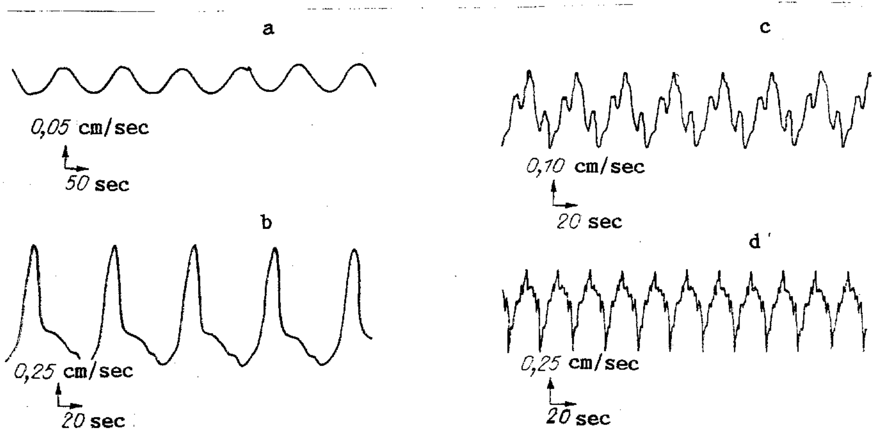

Fig. 4

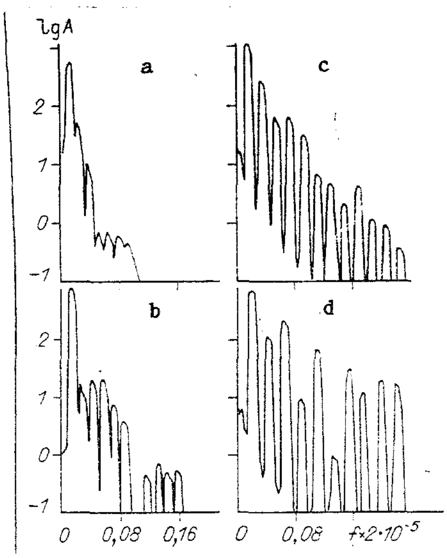

Fig. 5

Thus, the behavior of the functions $Re^*(\sigma_0)$, $Re_\lambda^*(\sigma_0)$, $f_L(\sigma_0)$, $f_h(\sigma_0)$, and $A^2(Re)$ in thick ($\sigma_0 \leq 497$) and thin ($\sigma_0 \geq 843$) layers differs from one another. Proceeding from qualitative comparisons, we note that as σ_0 makes the transition through the interval (497, 843) a change more noticeable than in the remaining region takes place in the shape and spectrum of the self-oscillations. This may be associated with the fact that the set of actively interacting modes changes as σ_0 changes. In all probability, with a reduction in the dimensionless depth of the layer it is the modes of a lesser scale that begin to play a greater role.

5. The Effect of Change in the Self-Oscillation Frequency with a Change in the Harmonic Coefficient. It becomes possible to observe this effect because the increment (decrements on reduction of Re, conversely) in the amplitude of the fundamental harmonics are greater than with the overtones. For example, in a flow with $\sigma_0 = 843$ by slow growth $Re = 1.17Re^*$ is established. Within approximately 0.5 h the excitation of self-oscillations is recorded. The increment in amplitude and self-oscillation frequency growth is equal: $\gamma = (5.1 \pm 0.2) \cdot 10^{-3}$ sec^{-1} and $f_L = 730 \pm 5$ ($T = 69.5 \pm 0.5$ sec). Within 1 h the oscillation amplitude is, for all intents and purposes, established, and its shape (initially, nearly sinusoidal) continues to change because of excitation and development of higher harmonics. The relative part of the energy contained within these is raised; frequency drops and is equal to 710 ± 3 ($T = 70.4 \pm 0.3$ sec). Finally, the amplitude and shape of the self-oscillations becomes established within 2 h, and here $f_L = 697 \pm 2$ ($T = 71.7 \pm 0.2$ sec). Thus, on excitation of the higher harmonics the self-oscillation frequency diminishes. There is also the opposite effect, namely, the increasing frequency as the harmonics become attenuated, or in the case of a diminution in the clear factor. In weakly supercritical flow in which the value of the clear factor (and consequently, a change in frequency) is small, no change can be observed within this factor within the limits of measurement error.

This effect apparently explains the fact as to why the relative change in frequency becomes smaller, the larger σ_0, given the identical change in Re/Re^* (see Sec. 4), since the change in the harmonic factor in this case is correspondingly greater. It is impossible to carry out any quantitative estimates on the basis of the existing results.

The relationship for the change in the frequency of the self-oscillations on excitation of the overtones was first derived by Van der Pol [13]. It was demonstrated in [3] that there exists a reduction in the spatial frequency as well (an increase in the dimensions of the vortices generated on loss of fluid stability, where this fluid is rotating within an ellipsoidal rhythm) in the excitation of spatial harmonics.

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CALCULATING THE NONISOTHERMAL SEPARATION STREAMLINING OF A SPHERE

K. B. Koshelev and M. P. Strongin

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Problems in technology frequently deal with the determination of resistance and heat-exchange factors for a solitary sphere, where its temperature is significantly different from that of the incoming flow of gas. In chemically reactive systems, moreover, the need arises for detailed knowledge of the fields of velocity and temperature in the flow about the particle.

A considerable number of studies (for example, [1-4]) has been devoted to the streamlining of a sphere by a uniform incompressible steady flow. These studies have enabled us to ascertain a detailed pattern of flow, coincident with experiment in such minute parameters as the angle of vortex separation and the length of the recirculation zone behind the trailing edge. Attempts have recently been made to calculate the nonisothermal problem [5], as well as the problem of the streamlining of a vaporized droplet in the case of small mass-exchange coefficients [6]. There exists a large quantity of work on the supersonic streamlining of a sphere at large Reynolds numbers Re_∞, a substantial portion of which is covered in [7, 8]. Hypersonic streamlining of a sphere at moderate values of Re_∞ is dealt with in [9], but these calculations are methodological in nature, owing to the fact that for description of the gas flow at the Reynolds and Mach numbers under consideration, when the Knudsen numbers $Kn = M_\infty/Re_\infty > 0.1$, and the Navier—Stokes equations, are, generally speaking, inapplicable. In studies dealing with the supersonic streamlining of a sphere, the authors have generally been interested in the characteristics of the flow in the forward part

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